Chapter 1 to 6 test

/42 marks

1. **DO NOT USE A CALCULATOR IN THIS QUESTION.**

The polynomial $p(x) = 2x^3 - 3x^2 + qx + 56$ has a factor $x - 2$.

a) Show that $q = -30$.

[1]

$p(2) = 16 - 12 + 2q + 56$

$0 = 60 + 2q$

$-60 = 2q$

$q = -30$ (shown)

b) Factorise p(x) completely and hence state all the solutions of p(x) = 0.

[4]

$$\begin{array}{r|l} & 2x^2 + x - 28 \\ \hline x-2 & 2x^3 - 3x^2 - 30x + 56 \\ & -(\cancel{2x^3} - 4x^2) \\ \hline & \cancel{x^2} - 30x \\ & -(\cancel{x^2} - 2x) \\ \hline & -28x + 56 \\ & -28x + 56 \\ \hline & 0 \end{array}$$

$p(x) = (x-2)(2x^2 + x - 28)$

$= (x-2)(2x-7)(x+4)$

$\therefore x = 2$ or $x = \frac{7}{2}$ or $x = -4$

2. (a) Express $12x^2 - 6x + 5$ in the form $p(x-q)^2 + r$, where *p*, *q* and *r* are constants to be found.

[3]

$$12x^2 - 6x + 5 = p(x^2 - 2qx + q^2) + r$$
$$= px^2 - 2pqx + pq^2 + r$$

$p = 12$

$-2pq = -6$

$24q = 6$

$q = \frac{1}{4}$

$pq^2 + r = 5$

$12 \times \frac{1}{16} + r = 5$

$\frac{3}{4} + r = 5$

$r = \frac{20-3}{4}$

$= \frac{17}{4}$

$$\therefore 12x^2 - 6x + 5 = 12\left(x - \frac{1}{4}\right)^2 + \frac{17}{4}$$

(b) Hence find the greatest value of $(12x^2 - 6x + 5)^{-1}$ and state the value of *x* at which this occurs.

[2]

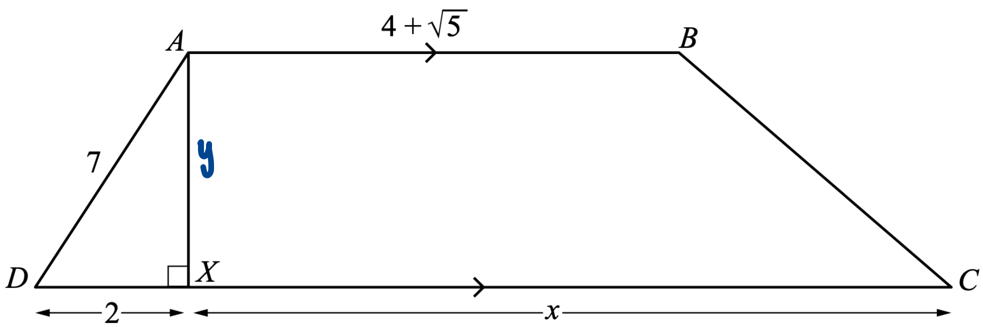
$$\frac{1}{12x^2 - 6x + 5}$$

$(1/4, 4/17)$

greatest value $= 4/17$

value of $x = 1/4$

3. **DO NOT USE A CALCULATOR IN THIS QUESTION.**



The diagram shows a trapezium *ABCD* in which *AD* = 7 cm and *AB* = (4 + $\sqrt{5}$) cm. *AX* is perpendicular to *DC* with *DX* = 2 cm and *XC* = *x* cm.

Given that the area of trapezium *ABCD* is 15($\sqrt{5}$ + 2)cm^2, obtain an expression for *x* in the form *a* + *b*$\sqrt{5}$, where *a* and *b* are integers.

[6]

$y^2 = 49 - 4$

$= 45$

$y = 3\sqrt{5}$

$\text{area} = \frac{1}{2}(a+b)h$

$15(\sqrt{5}+2) = \frac{1}{2}(4+\sqrt{5}+2+x)(3\sqrt{5})$

$30(\sqrt{5}+2) = (6+\sqrt{5}+x)3\sqrt{5}$

$30\sqrt{5}+60 = 18\sqrt{5}+15+3x\sqrt{5}$

$12\sqrt{5}+45 = 3x\sqrt{5}$

$x\sqrt{5} = 4\sqrt{5}+15$

$x = \frac{4\sqrt{5}+15}{\sqrt{5}}$

$= \frac{4\sqrt{5}+15(\sqrt{5})}{5} = \frac{20+15\sqrt{5}}{5}$

$= 4+3\sqrt{5}$

4. (a) On the axes below, sketch the graph of $y = |2x + 5|$ and the graph of $y = |2 - x|$, stating the coordinates of the points where each graph meets the coordinate axes.

[4]

$y = |2x+5|$

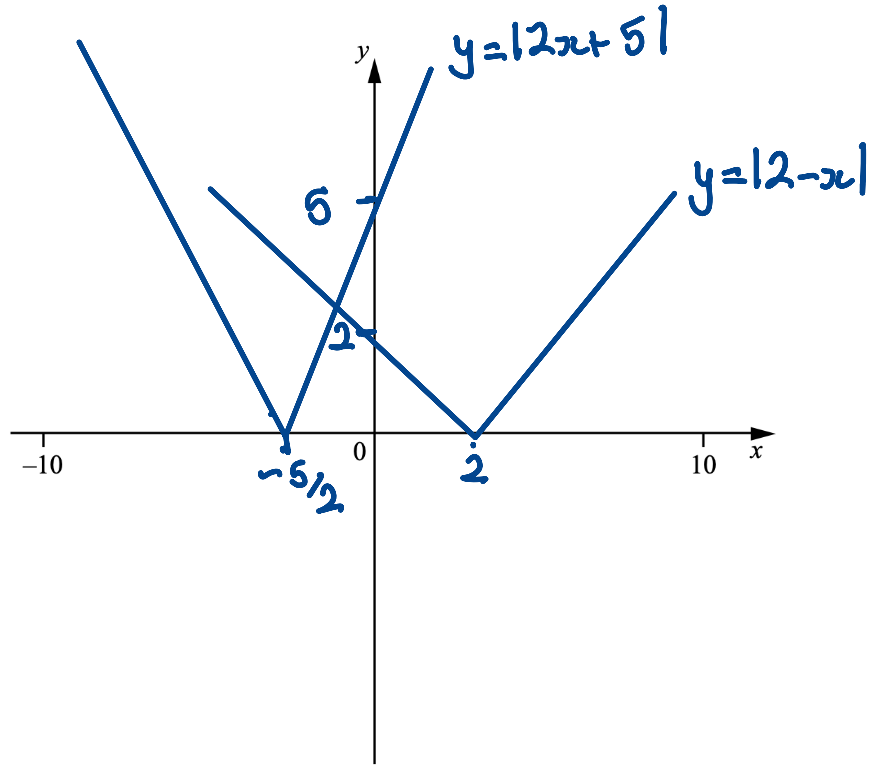
$x = 0, y = 5$

$y = 0, x = -5/2$

$y = |2-x|$

$x = 0, y = 2$

$y = 0, x = 2$



(b) Solve $|2x + 5| \leq |2 - x|$.

[3]

$(2x+5)^2 \leq (2-x)^2$

$4x^2 + 20x + 25 \leq 4 - 4x + x^2$

$3x^2 + 24x + 21 \leq 0$

$x^2 + 8x + 7 \leq 0$

$x^2 + x + 7x + 7 \leq 0$

$x(x+1) + 7(x+1) \leq 0$

$(x+1)(x+7) \leq 0$

$x = -1$ or -7

$-7 \leq x \leq -1$

5. Solve

$$xy = 3$$
$$x^4y^5 = 486$$

[3]

$xy = 3$

$y = 3/x$

$x^4y^5 = 486$

$x^4(3/x)^5 = 486$

$x^4\left(\frac{243}{x^5}\right) = 486$

$\frac{243}{x} = 486$

$243 = 486x$

$x = 1/2 \quad \therefore y = 3/x = 3/\tfrac{1}{2}$

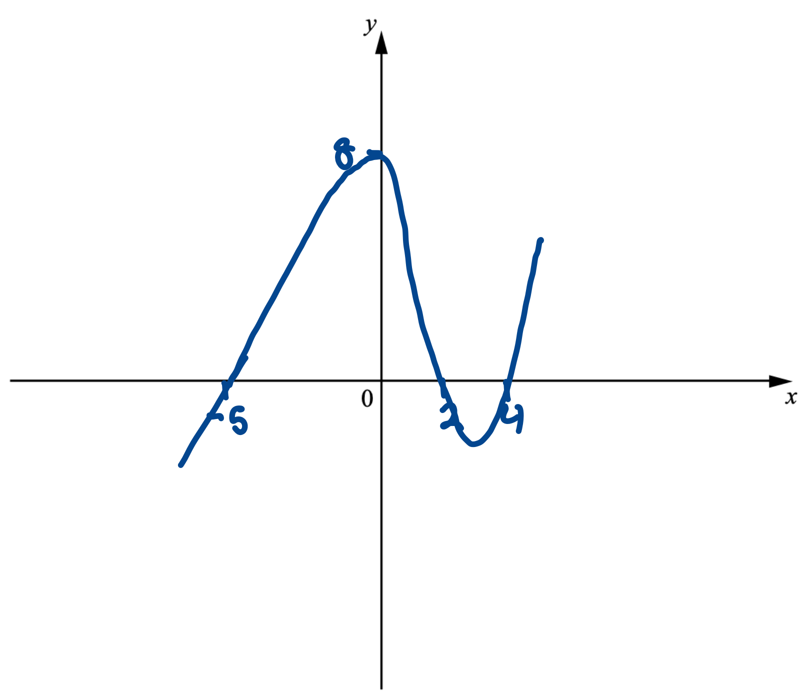
$= 6$

6. (a) On the axes below, sketch the graph of $y = \frac{1}{5}(x-2)(x-4)(x+5)$, showing the coordinates of the points where the graph meets the coordinate axes.

[2]

$y = 0,\ x = 2, 4$ or -5

$x = 0,\ y = 8$



(b) Hence solve $(x - 2)(x - 4)(x + 5) \leq 0$.

$2 \leq x \leq 4 \quad x \leq -5$

[1]

7. Functions g and h are such that

$g(x) = 2 + 4\,lnx$ for $x > 0$,

$h(x) = x^2 + 4$ for $x > 0$.

(a) Find $g^{-1}(x)$.

[4]

$x = 2 + 4\ln y$

$x - 2 = 4\ln y$

$\frac{x-2}{4} = \ln y$

$y = e^{\frac{x-2}{4}}$

$g^{-1}(x) = e^{\frac{x-2}{4}}$

(b) Solve $gh(x) = 10$.

[3]

$2 + 4\ln(x^2+4) = 10$

$4\ln(x^2+4) = 8$

$\ln(x^2+4) = 2$

$x^2 + 4 = e^2$

$x^2 = e^2 - 4$

$x = \sqrt{e^2 - 4}$

$= 1.84$

8. (a) Simplify $log_a\sqrt{2} + log_a 8 + log_a(\frac{1}{2})$, giving your answer in the form $p\, log_a 2$, where p is a constant.

[2]

$\frac{1}{2}\log_a 2 + 3\log_a 2 - \log_a 2$

$= \log_a 2^{7/2} - \log_a 2$

$= \log_a 2^{5/2}$

$= \frac{5}{2}\log_a 2$

(b) Solve the equation $log_3 x - log_9 4x = 1$.

[4]

$\log_{9^{1/2}} x - \log_9 4x = 1$

$2\log_9 x - \log_9 4x = \log_9 9$

$\log_9 x^2 - \log_9 4x = \log_9 9$

$\frac{x^2}{4x} = 9$

$x^2 = 36x$

$x^2 - 36x = 0$

$x(x-36) = 0$

$x = 0$ or 36

(reject 0)

$x = 36$